The second set can, of course, be written down from the first set by writing $\pi-A$ for $a$, etc., so that we need remember only the sets (1), (3), and (4). To determine what formula to use in any case, we notice that set (1) connects *three sides and an angle;* set (2), *three angles and a side;* set (3), *two angles and two sides, one of each being included;* set (4), *two angles and two opposite sides.*

We, therefore, choose at once the formula connecting the three given and the one required quantity. In the case of right, or of quadrantal, triangles, one term will always vanish, thus giving the required formula in logarithmic form.

---

These formulae must sooner or later be learned by the student of spherical trigonometry, and to apply the correct one in any case is easy.

But as an aid in remembering them, we may note that (1) and (3) begin with a *side* and end with the *opposite angle*, the converse being the case in (2). Also in (3) the sides and angles occur in the easily remembered form

$$a,\ b,\ b,\ C,\ C,\ A;$$

and the respective functions occur in the symmetric form

$$\cot,\ \sin,\ \cos,\ \cos,\ \sin,\ \cot.$$

This form of writing (3) I received from the astronomical lectures of Prof. H. H. Turner, of Oxford, and I have found it a very useful way of remembering it.

---

I think the deduction of the right and quadrantal triangle formulae from these general formulae is economical, and have found it easier to apply them to right triangles than to use the regular formulae by the means of Napier's rules.

They also apply equally well to the quadrantal triangles which are of such frequent occurence in the usual application of spherical trigonometry to elementary astronomy.

# DEPARTMENTS.

## SOLUTIONS OF PROBLEMS.

### ARITHMETIC.

**166. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, Ohio.**

If I sell one of my farms for $\$A$,=\$4500, and the other for $\$B$,=\$1800, I will gain $p\%$, =5%, on cost of both; but if I sell the dearer farm for $\$C$,=\$4000, and the other at cost, I will lose $p\%$,=5%. Find the cost of each farm.

Solution by G. B. M. ZERR, A. M., Ph.D., The Temple College, Philadelphia, Pa., and J. E. SANDERS, Hackney, Ohio.

$$\frac{\$A+\$B}{1+p}=\frac{\$6300}{1.05}=\$6000=\text{cost of both farms.}$$

$$\$A+\$B-\frac{\$A+\$B}{1+p}=\frac{\$p(A+B)}{1+p}=\frac{\$6300\times.05}{1.05}=\$300=\text{gain.}$$

$$C+\frac{p(A+B)}{1+p}=\frac{C+p(A+B+C)}{1+p}=\$4000+\$300=\$4300=\text{cost of dearer farm.}$$

$$\frac{A+B}{1+p}-\frac{C+p(A+B+C)}{1+p}=\frac{(A+B)(1-p)}{1+p}-C=\$6000-\$4300=\$1700$$

$=$cost of cheaper farm.

Also solved in a similar manner and with same result by *G. W. GREENWOOD.*

## ALGEBRA.

171. Proposed by IDA M. SCHOTTENFELTZ, A. M., New York, N. Y.

$ay^2+a=bxy+cx$, $bx^2+b=axy+cy$. Solve for $x$ and $y$.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

$ay^2+a=bxy+cx\ldots.(1)$. $bx^2+b=axy+cy\ldots.(2)$.

From (1), $x=a(y^2+1)/(by+c)\ldots.(3)$.

(3) in (2) gives $[(a^2+b^2)y^2+2bcy+a^2+c^2](cy-b)=0$.

$$\therefore\ y=b/c,\ y=-\frac{1}{a^2+b^2}\{bc\mp a\sqrt{[-(a^2+b^2+c^2)]}\}.$$

$$x=a/c,\ x=-\frac{1}{a^2+b^2}\{ac\pm b\sqrt{[-(a^2+b^2+c^2)]}\}.$$

Also solved by *MARCUS BAKER.*

## GEOMETRY.

193. Proposed by PROFESSOR BEYENS.

Si le rapport du segment d'une base de la sphère à l'hemisphère est $m/n$, le rapport de l'hauteur du segment à deux bases qui resultera au rayon est égal à $2\sin\frac{1}{3}[\sin^{-1}(n-m)/n]$. [Problem 9699, *Educational Times.*]

Solution by J. R. HITT, Goss, Miss.; G. B. M. ZERR, A. M., Ph. D., The Temple College, Philadelphia, Penn., and G. W. GREENWOOD, B. A., Professor of Mathematics and Astronomy, McKendree College, Lebanon, Ill.

Let $R$ denote radius of sphere, $h$ the altitude of segment of two bases, $R-h$=altitude of segment of one base. Then, $\pi(R-h)^2[R-\frac{1}{3}(R-h)]/\frac{2}{3}\pi R^3$